Unexplored regions in teleparallel gravity : sign changing dark energy density

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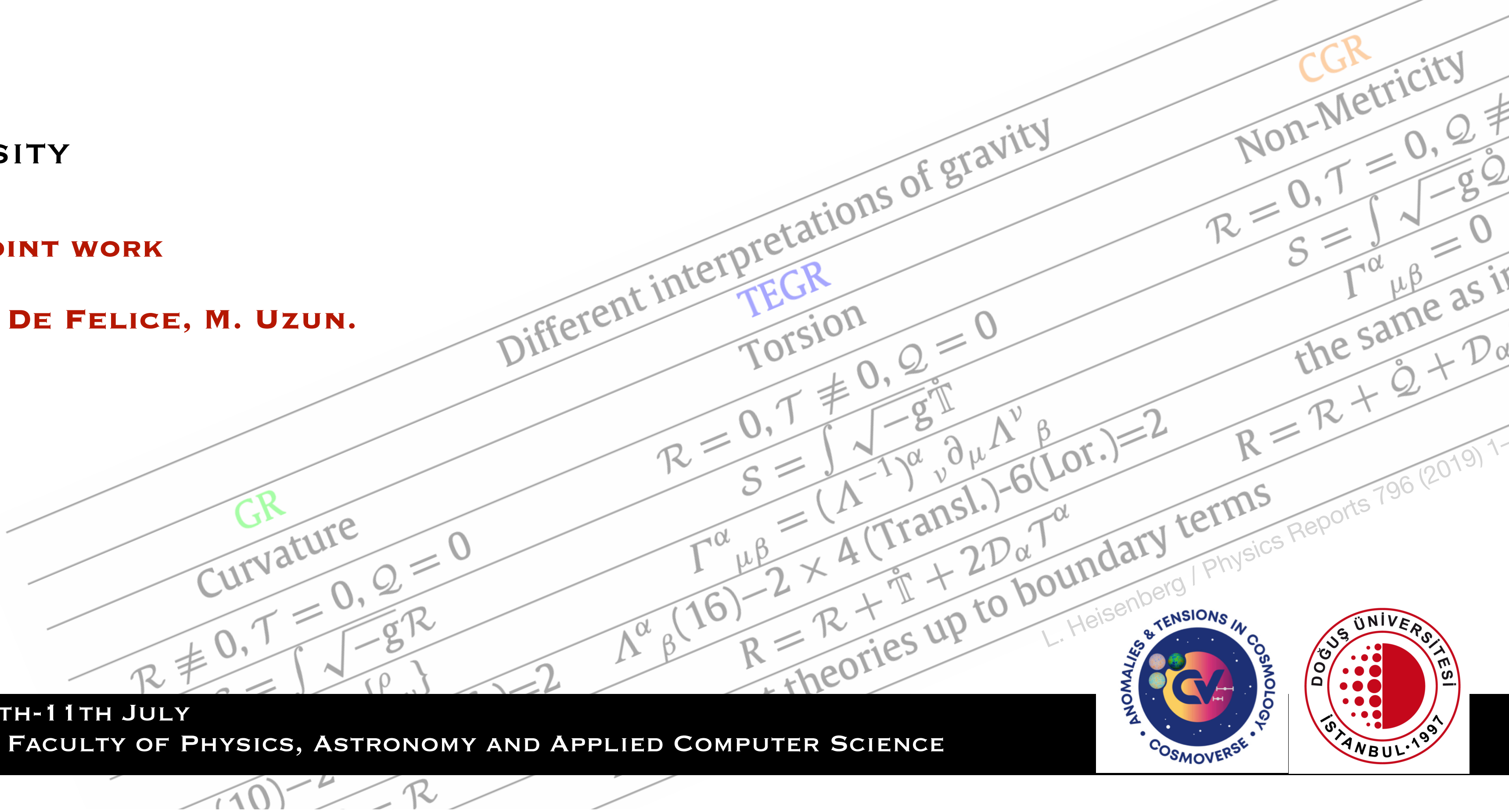
based on the ongoing joint work in collaboration with Ö.Akarsu, B. Bulduk, A. De Felice, M. Uzun.



European Cooperation in Science & Technology

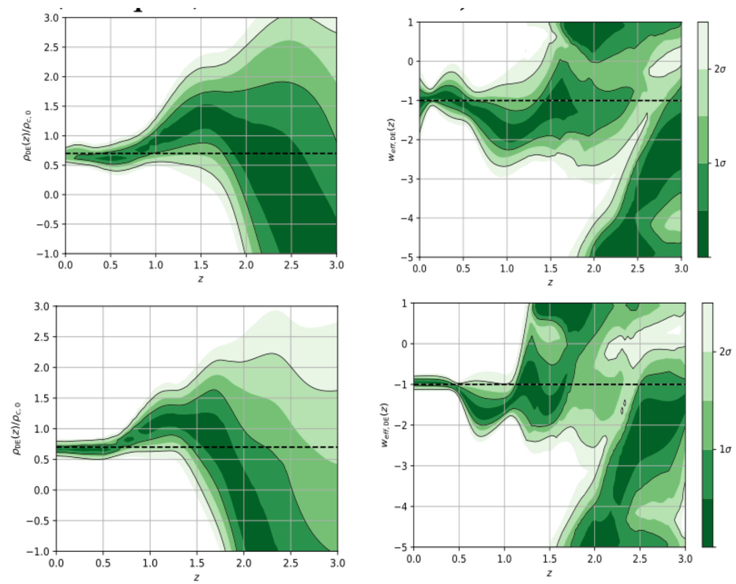
CosmoVerse@Kraków 2024, 9th-11th July
The Jagiellonian University, Faculty of Physics, Astronomy and Applied Computer Science

THE H0 TENSION— AS WELL AS A NUMBER OF OTHER LOW-REDSHIFT DISCREPANCIES— MAY BE ALLEVIATED

BY A DYNAMICAL DARK ENERGY THAT ASSUMES NEGATIVE OR VANISHING ENERGY DENSITY VALUES AT HIGH REDSHIFTS.

AKARSU,NK, KUMAR, NUNES, OZTURK, SHARMA, EPJC 80, 1050 (2020). 2004.04074

AKARSU KUMAR VAZQUEZ YADAV PHYS. REV. D 108, 023513 (2023), 2211.05742



MODEL INDEPENDENT/NON-PARAMETRIC OBSERVATIONAL RECONSTRUCTION OF DARK ENERGY STUDIES PREDICT DARK ENERGY WHICH ATTAINS NEGATIVE VALUES IN THE PAST ACCOMPANIED BY A SINGULAR EOS PARAMETER AS WELL.

CALDERO´N ET.AL, PRD 103, 023526 (2021) 2008.10237,

ESCAMILLA, AKARSU, DI VALENTINO & VAZQUEZ, JCAP 11, 051 (2023). 2305.16290 ,

SABOGAL, AKARSU, BONILLA, DI VALENTINO, NUNES, 2407.04223

THESE STUDIES SHOW THAT SOME NON-TRIVIAL FUNCTIONS/BEHAVIORS ARE REQUIRED TO ALLEVIATE TENSIONS.

WE EXPLORE VIABLE COSMOLOGIES IN A PARTICULAR MODEL DUBBED EXPONENTIAL INFRARED TELEPARALLEL GRAVITY IN A THEORETICAL FRAMEWORK, SINCE IT ALLOWS A TRANSITION TO NEGATIVE ENERGY DENSITIES THROUGH TORSIONAL DARK ENERGY.

$$f(T) = Te^{\beta T_0/T} \qquad \rho_{\rm T}(z=z_\dagger)=0,$$

EVEN WE THEORETICALLY EXPLORE THE UNCHARTED REGIONS OF THIS MODEL FOR THE CASE PREDICTING TORSIONAL DARK ENERGY FEATURING PHANTOM BEHAVIOUR.

THE OTHER SOLUTION BRANCH YIELDING β_- TO BE OVERLOOKED SO FAR IN THE LITERATURE.

Why β < 0 case?

- f(T) gravity is capable on the way to integrating Λ_sCDM into a theoretical framework.

- Λ_sCDM is the most economical phenomenological extension of the standard ΛCDM,

- its capability to simultaneously resolve the major cosmological tensions, notably, H0, S8 and Mb tensions along with less significant tensions currently present in ΛCDM.

Akarsu et.al. 2307.10899

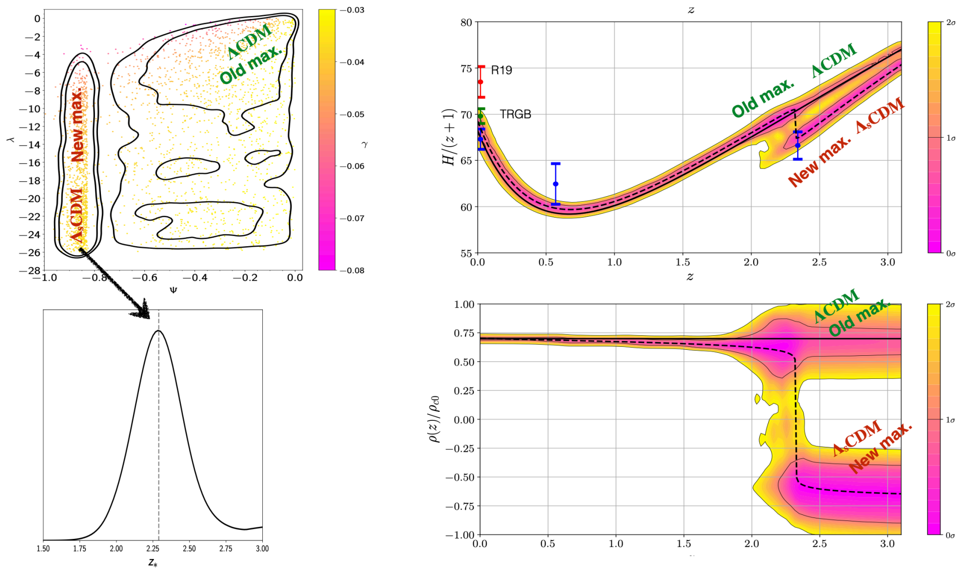
- the model comprises a mirror AdS-dS transition, realized by a sign-switching cosmological constant represented as

$\Lambda = \Lambda_{s0}\,\mathrm{sgn}[z_\dagger - z]$

- where transition occurring at the redshift $z_\dagger \sim 2$ on average.

- The shift from a negative value to a positive value is unconventional and challenging in the achieving of a concrete mechanism underlying the Λ CDM model from fundamental theories of physics.

- Nevertheless, successes on alleviating tensions are quite encouraging for the search of incorporating this model into a well established and predictive theoretical framework.



Akarsu barrow Escamilla Vazquez Phys. Rev. D 101, 063528 (2020), 1912.08751
Akarsu Özülker kumar Vazquez Phys. Rev. D 104, 123512 (2021),2108.09239

Akarsu kumar Vazquez Yadav Phys. Rev. D 108, 023513 (2023), 2211.05742

$$\partial_t^2 \delta_{\rm m} = -2H\partial_t\delta_{\rm m} + 4\pi G\bar{\rho}_{\rm m}\delta_{\rm m},$$

Nyugen, Huterer, Wen,

Evidence for Suppression of Structure Growth in the Concordance Cosmological Model,
Phys.Rev.Lett. 131 (2023) 11, 111001

Possible scenarios

- Overall sign change of the metric [alexandre, gielen, magueijo, JCAP 02 (2024) 036, 230611502] : imaginary space" extensions of the usual Lorentzian theory, with $a^2 < 0.$

$$\tilde{\Lambda} = \mathrm{sgn}(\tilde{a}^2)\Lambda$$

- a string inspired model where Casimir forces of fields inhabiting the bulk of the dark dimension scenario [Anchordoqui,antoniadis,lüst, Phys. Lett. B 855 (2024) 138775,2312.12352, &noble, soriano 2404.17334
- Type II minimally modified gravity (VCDM). [akarsu et al. 2402.07716, 2406.07526]
- Construction of Λ_sCDM from the presence of higher dimensions [akarsu, bulduk, nk, özülker,Perivolaropoulos]

$$\tilde{\chi} = \tilde{\Lambda} - \frac{n(n-1)}{2}\frac{k_{\mathrm{int}}}{s^2}$$

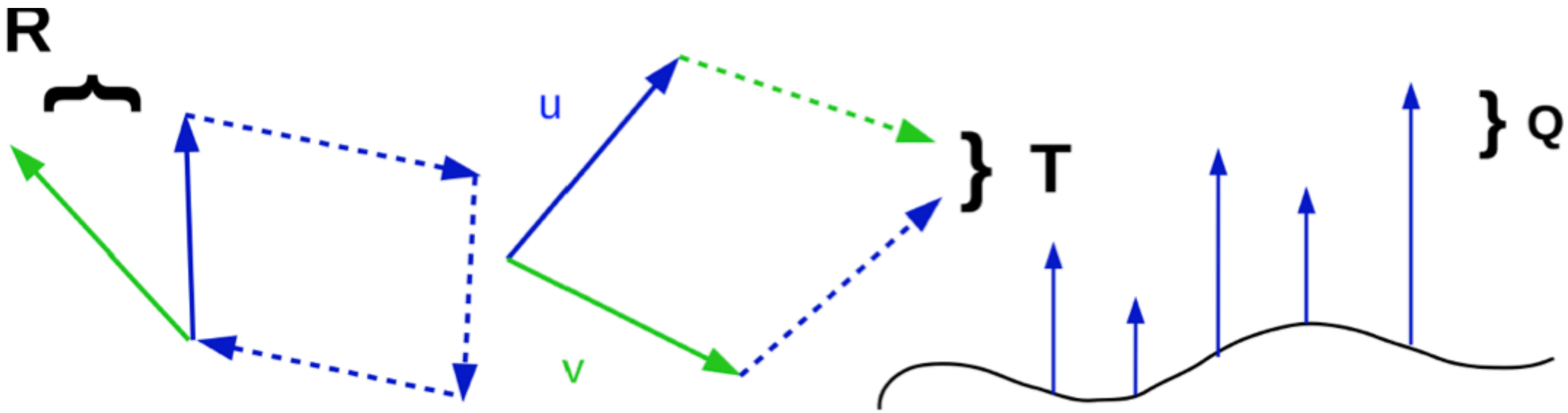
Can they be distinguishable from different predictions from ΛCDM and from each others?

Previous attempts satisfying negative energy densities

- Brans dicke extension of GR [akarsu, nk, Özdemir, vazquez EPJC 80 (2020) 32, 1903.06679]
- Rastall extension of gr [Akarsu,NK, Kumar, Nunes, ozturk, Sharma, EPJC 80, 1050 (2020). 2004.04074]
- Simple graduated dark energy and spatial curvature, Acquaviva, akarsu, nk, Vazquez, PRD 104, 023505 (2021), 2104.02623

A schematic illustration of the roles of curvature $R^{\alpha}{}_{\beta\mu\nu}$, torsion $T^{\alpha}{}_{\mu\nu}$ and non-metricity $Q_{\alpha\mu\nu}$

L. Heisenberg / Physics Reports 796 (2019) 1–113



Curvature (Rotation) | Torsion (non closing) | Non metricity (changing lengths)

If the space–time contains curvature, then the direction of a vector field changes when we move it along a closed circle

In the presence of torsion, parallelograms do not close.

non-metricity changes the norm of vector fields transported along a curve

Teleparallel gravity : $R^{\alpha}{}_{\beta\mu\nu} = 0$, $Q_{\alpha\mu\nu} = 0$ and $T^{\alpha}{}_{\mu\nu} \neq 0$

TELEPARALLEL DESCRIPTION OF GRAVITY

TELEPARALLEL GRAVITY: FROM THEORY TO COSMOLOGY
BAHAMONDE ET.AL REP. PROG. PHYS. 86 026901 (2023)
2106.13793

a 4-dimensional differentiable manifold whose tangent space is, at each point, a Minkowski spacetime.

e^a_μ four linear independent vector fields (vierbeins/tetrads) defined on this smooth manifold M.

To obtain a nondegenerate metric, the vierbeins should satisfy the orthonormality conditions:

$e^\mu_a e^a_\nu = \delta^\mu_\nu$ and $e^\mu_a e^b_\mu = \delta^b_a$.

The Lorentzian metric tensor of the spacetime can be derived as $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$ where $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$

torsionless Levi- Civita connection → curvatureless Weitzenböck connection, which is defined by the vierbeins in the following way:

$$\Gamma^\sigma{}_{\mu\nu} = e_a{}^\sigma \partial_\nu e^a{}_\mu = -e^a{}_\mu \partial_\nu e_a{}^\sigma,$$

whose nonsymmetric feature gives rise to the definition of torsion tensor as follows:

$$T^\sigma{}_{\mu\nu} = \Gamma^\sigma{}_{\nu\mu} - \Gamma^\sigma{}_{\mu\nu} = e_a{}^\sigma \left(\partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu\right)$$

The spacetime-indexed superpotential tensor is defined as $S_\sigma{}^{\mu\nu} = \frac{1}{4}\left(T^{\nu\mu}{}_\sigma + T_\sigma{}^{\mu\nu} - T^{\mu\nu}{}_\sigma\right) + \frac{1}{2}\left(\delta^\mu_\sigma T^{\lambda\nu}{}_\lambda - \delta^\nu_\sigma T^{\lambda\mu}{}_\lambda\right)$

(skew symmetric in the last pair of indices)

Contracting it with the torsion tensor yields the torsion scalar (Weitzenböck invariant), viz., $T = S_\sigma{}^{\mu\nu} T^\sigma{}_{\mu\nu}$

$$T = \frac{1}{4} T^\sigma{}_{\mu\nu} T_\sigma{}^{\mu\nu} + \frac{1}{2} T^\sigma{}_{\mu\nu} T^{\nu\mu}{}_\sigma - T^\sigma{}_{\mu\sigma} T^{\nu\mu}{}_\nu$$

F(T) GRAVITY AND COSMOLOGY

The action reads

$$\mathcal{S} = \int \mathrm{d}^4x \, \det(e^a_\mu) \left[\frac{1}{2\kappa} f(T) + \mathcal{L}_\mathrm{m}\right]$$

$$\det(e^a_\mu) \rightarrow \sqrt{-g}$$

$$\frac{1}{\det(e^b{}_\lambda)} f_T \partial_\mu \left[\det(e^b{}_\lambda) e^\sigma{}_a S_\sigma{}^{\mu\nu}\right] + f_{TT} e^\sigma{}_a S_\sigma{}^{\mu\nu} \partial_\mu T$$
$$- f_T e^\lambda{}_a T^\sigma{}_{\mu\lambda} S_\sigma{}^{\nu\mu} + \frac{1}{4} f e^\nu{}_a = \frac{1}{2}\kappa e^\sigma{}_a \Theta_\sigma{}^\nu,$$

$$\Theta_\mu^{\ \nu} = e^a{}_\mu \left[-\frac{1}{\det(e^b_\lambda)} \frac{\delta \mathcal{L}_\mathrm{m}}{\delta e^a_\nu}\right]$$

$$\Theta_\mu^{\ \nu} = \mathrm{diag}[-\rho, p, p, p]$$

we proceed by assuming the following vierbeins

to investigate the cosmological applications of the model

$$e_\mu{}^a = \mathrm{diag}(1, a(t), a(t), a(t)),$$

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) \qquad T = 6H^2,$$

$$3H^2 - \frac{1}{2}\left(T + f - 2T f_T\right) = \kappa\rho,$$

$$-2\dot{H} - 3H^2 - \frac{1}{2}\left(\frac{T f_T - f - 2T^2 f_{TT}}{f_T + 2T f_{TT}}\right) = \kappa p,$$

$$f_T = \frac{\mathrm{d}f}{\mathrm{d}T} \qquad f_{TT} = \frac{\mathrm{d}^2 f}{\mathrm{d}T^2}$$

EXPONENTIAL INFRARED TELEPARALLEL GRAVITY $f(T) = Te^{\beta T_0/T}$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = 0,$$

BASED ON SIX PARAMETERS LIKE THE STANDARD ΛCDM SUCH THAT B IS NOT A FREE PARAMETER/ IS DETERMINED BY THE PRESENT-DAY ENERGY DENSITY OF MATTER (ΩMO) IN THE CONTEXT OF FLRW COSMOLOGY.

AWAD, EL HANAFY, NASHED & SARIDAKIS, JCAP 02, 052 (2018). 1710.10194

HASHIM, EL HANAFY, GOLOVNEV&EL- ZANT, JCAP 07, 052 (2021). 2010.14964 JCAP 07, 053 (2021). 2104.08311

$$\left(\frac{H^2}{H_0^2} - 2\beta\right) e^{\beta H_0^2/H^2} = \Omega_{\rm m0}(\beta)\,(1+z)^3,$$

$$\Omega_{\rm m0}(\beta) = (1-2\beta)e^{\beta}\,,$$

- THE $\beta = 0$ CASE IS TELEPARALLEL EQUIVALENT OF GENERAL RELATIVITY (TEGR) GIVING RISE TO EINSTEIN-DE SITTER UNIVERSE WITH ΩMO = 1.
- THE STUDIES SO FAR ADHERED TO THE POSITIVE POWER OF EXPONENTIAL, VIZ., $\beta > 0$ CASE, EXCLUDING NEGATIVE ONE, AND OBTAINED AN EFFECTIVE DE WHOSE DENSITY PARAMETER IS BELOW THE PHANTOM DIVIDE LINE.
- HOWEVER, WE WILL SHOW THAT $\beta < 0$ CASE CONVERSELY GENERATES AN EFFECTIVE DE WHOSE ENERGY DENSITY CHANGES SIGN AT A CERTAIN REDSHIFT.
- $\beta < 0$ IS A SUFFICIENT CONDITION TO AVOID INSTABILITIES/GHOSTS, YOU HAVE $F_{\rm T} > 0$ INDEPENDENTLY OF THE VALUE DYNAMICS OF T ON ANY BACKGROUND, INCLUDING THE FLRW SPACETIME, WHICH MAY BE SEEN CONVERSELY, WHEN B > 0, ONE NEEDS TO ENSURE THAT DYNAMICALLY THE UNIVERSE NEVER ENTERED THROUGH AN ERA DURING WHICH FT < 0.

$$f_T = e^{\beta T_0/T}\,(1-\beta T_0/T)\,,$$
$$= e^{\beta H_0^2/H^2}\,\left(1-\beta H_0^2/H^2\right),$$

$$\Omega_{\rm m0} = \kappa\rho_{\rm m0}/(3H_0^2)$$
$$\Omega = \rho/\rho_{\rm cr} \text{ with } \rho_{\rm cr} = 3H^2/\kappa$$

SEE FOR VARIOUS RECONSTRUCTION METHODS [DENT,DUTTA & SARIDAKIS, JCAP 01, 009 (2011). 1010.2215, DAOUDA, RODRIGUES & HOUNDJO, EPJC 72, 1893 (2012). 1111.6575] EXPOSING THE EXPLICIT HUBBLE PARAMETER OF SEVERAL F(T) MODELS, THEY STILL DO NOT PROVIDE AN EQUIVALENT RESPONSE.
THE REDSHIFT AS A FUNCTION OF THE HUBBLE PARAMETER AS FOLLOWS:

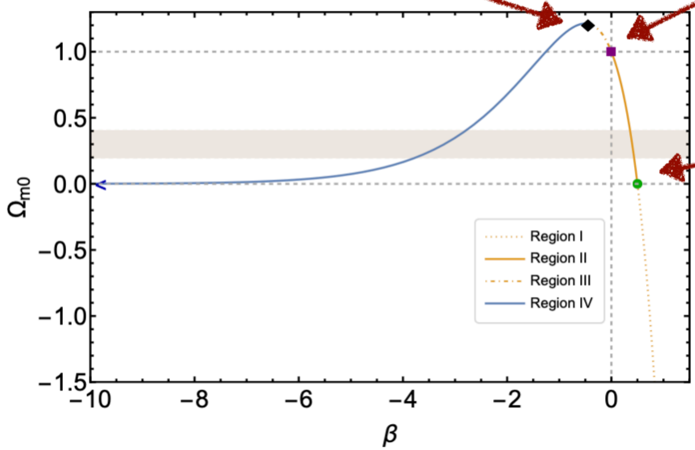
$$z(H) = e^{\frac{\beta}{3}(H_0^2/H^2-1)}\left[\frac{H^2/H_0^2 - 2\beta}{1-2\beta}\right]^{1/3} - 1, \qquad H^2 < 2\beta H_0^2.$$

Unexplored solution regions along with the known regions

$\Omega_{m0} = 2/\sqrt{e}$,

Point III $(\beta = -1/2)$:

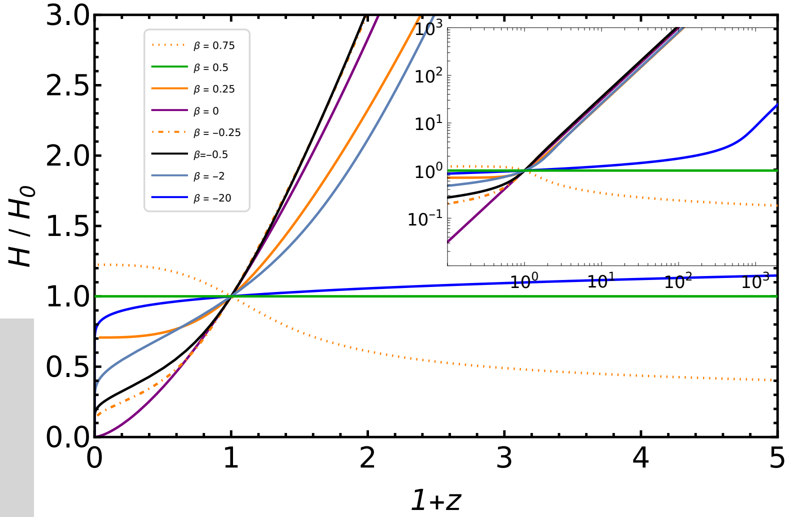
Point II $(\beta = 0 \to \Omega_{m0} = 1)$



MATTER DOMINATED TEGR, EINSTEIN-DE SITTER UNIVERSE.

Point I $(\beta = 1/2)$:

EMPTY UNIVERSE HOWEVER, A DE SITTER BACKGROUND $H^2 = H_0^2$ DUE TO THE F(T) MODIFICATION IN THE SPACETIME GEOMETRY EVEN THOUGH NO Λ IS INTRODUCED.

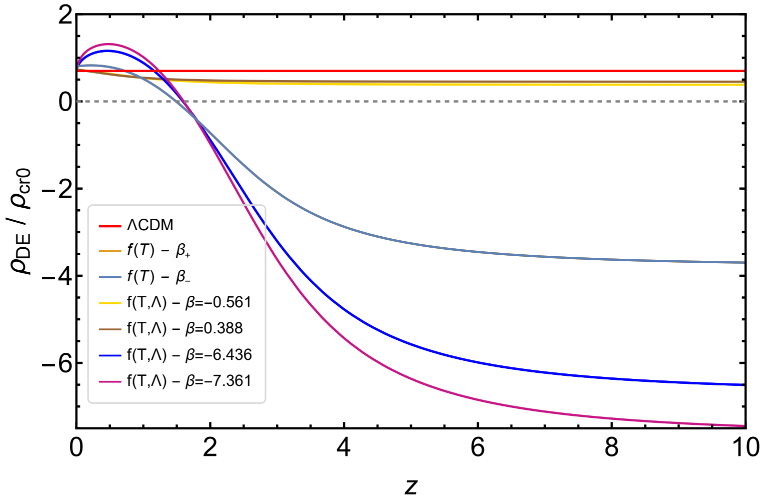


$$\left(\frac{H^2}{H_0^2} - 2\beta\right) e^{\beta H_0^2/H^2} = \Omega_{m0}(\beta)\,(1+z)^3,$$

$$\Omega_{m0}(\beta) = (1-2\beta)e^{\beta}\,,$$

$$3H^2 = \kappa\rho_{m0}(1+z)^3 + \kappa\rho_T,$$

$$\rho_T(H) = \frac{3H^2}{\kappa}\left[1 - \left(1 - 2\beta H_0^2/H^2\right) e^{\beta H_0^2/H^2}\right]$$



Region I $(\beta > 1/2)$: $\Omega_{m0} < 0$ $H^2 < 2\beta H_0^2$.

as $z \to \infty$, $H \to 0$, a Minkowski limit

as $z \to -1$, then $H^2 \to 2\beta H_0^2$

Region II $(0 < \beta < 1/2)$ $0 < \Omega_{m0} < 1$ $H^2 > 2\beta H_0^2$

AWAD, EL HANAFY, NASHED & SARIDAKIS, JCAP 02, 052 (2018). 1710.10194

HASHIM, EL HANAFY, GOLOVNEV & EL-ZANT, JCAP 07, 052 (2021). 2010.14964 JCAP 07, 053 (2021). 2104.08311

Region III $(-1/2 < \beta < 0)$: $1 < \Omega_{m0} < 2/\sqrt{e}$,

Region IV $(\beta < -1/2)$ $0 < \Omega_{m0} < 2/\sqrt{e}$.

New

THIS REGION ALSO INCLUDES THE OBSERVATIONALLY REASONABLE Ω_{m0} INTERVAL SHOWN BY WHEAT BAND, HOWEVER, IT IS OVERLOOKED IN COSMOLOGICAL ANALYSES IN THE LITERATURE TO DATE.

Quantitative analysis

$\dot{a} = H(z)/(1+z)$ (comoving Hubble parameter)

Fixing $\theta_* = r_*/D_M(z_*)$

Utilizing from ΛCDM Planck18 constrained strictly/ almost model independently through the measurement of Θ* ($100\Theta_* = 1.041085$)

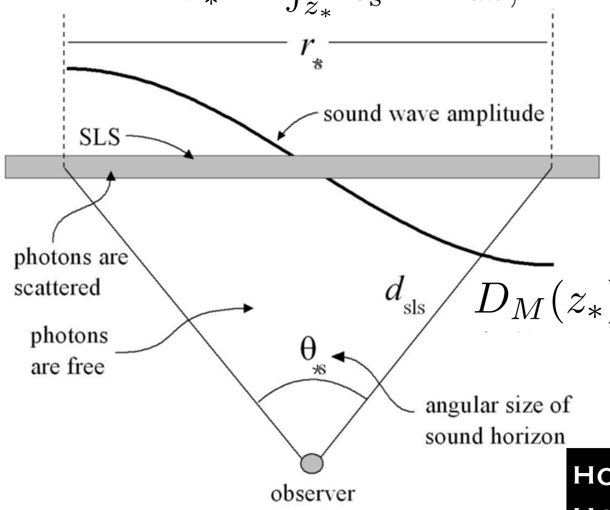
$\Omega_{m0}h^2 = 0.14314$

the CMB-based constraint

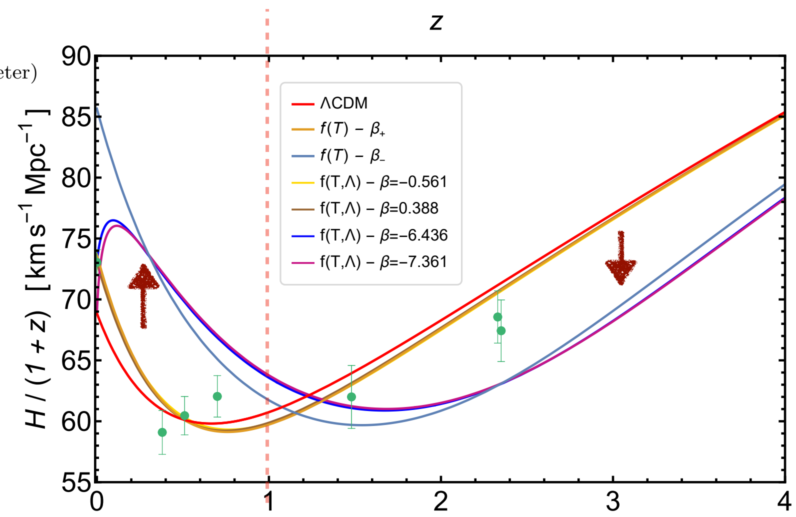
$D_M(z_*) = 13872.83$ Mpc

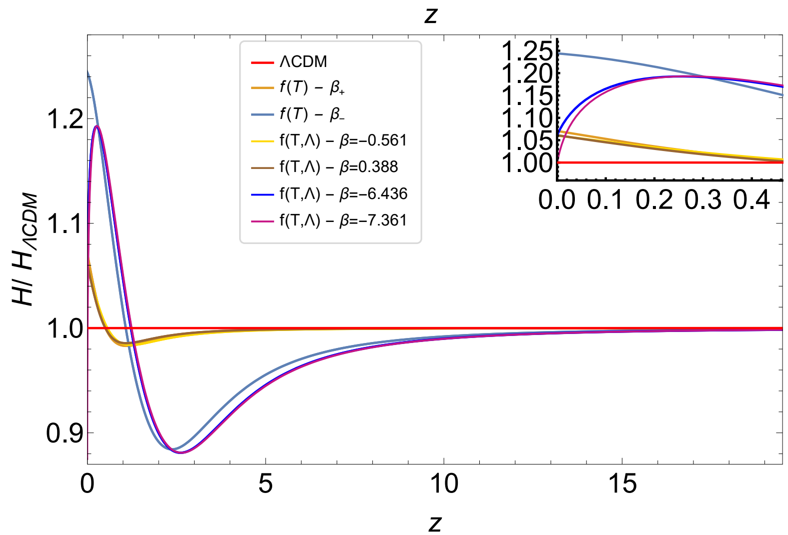
Effective at post-recombination epoch
No effect on pre-recombination epoch

$$r_* = \int_{z_*}^{\infty} c_s \breve{H}^{-1} dz,$$



$$D_M(z_*) = \int_0^{z_*} c H^{-1} dz.$$





Ho = 68.86 km s^{-1} Mpc^{-1}, Ωmo = 0.302 for ΛCDM model
Ho = 73.69 km s^{-1} Mpc^{-1}, Ωmo = 0.264 and β+ = 0.413 within Region II
Ho = 85.66 km s Mpc , Ωmo = 0.195 and β− = −3.782 within Region IV

h = Ho/(100 km s^{-1} Mpc^{-1})

EXPLORING THE VIABLE COSMOLOGIES BY INCLUDING Λ

$$f(T,\Lambda) = Te^{\beta T_0/T} + 2\Lambda,$$

On top of this interesting feature, we could still add to the f(T) a cosmological constant to have larger phenomenological possibilities. Accordingly, in Sec. V, we will widen the scope of the exploration for viable cosmologies by including Λ in the action while retaining the function that describes the exponential infrared teleparallel model.

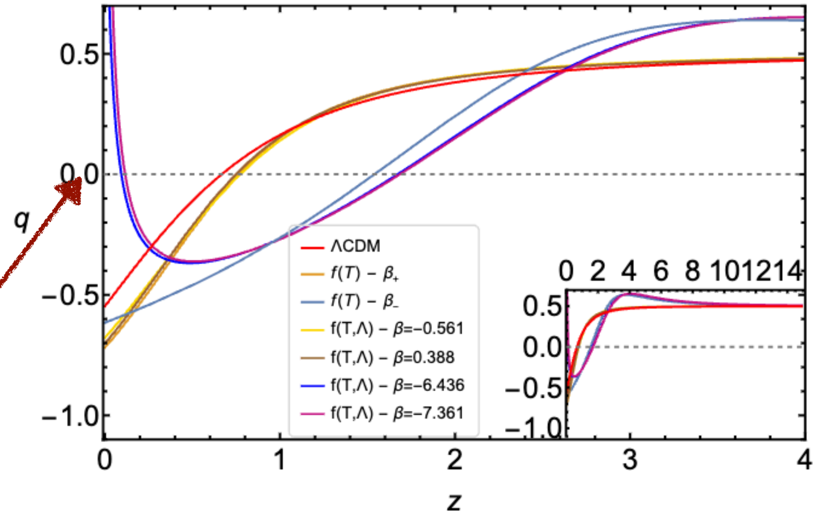
$$\rho_{\rm de} = \rho_{\rm T} + \rho_\Lambda,$$

$$\rho_{\rm T} = \frac{3H^2}{\kappa}\left[1-(1-2\beta H_0^2/H^2)e^{\beta H_0^2/H^2}\right],$$
$$\rho_\Lambda = \frac{\Lambda}{\kappa},$$

$$p_{\rm de} = p_{\rm T} + p_\Lambda,$$

$$p_{\rm T} = -\frac{\beta}{\kappa}\frac{3H_0^2(1+2\beta H_0^2/H^2)}{1-\beta H_0^2/H^2+2\beta^2 H_0^4/H^4},$$
$$p_\Lambda = -\frac{\Lambda}{\kappa}\frac{e^{-\beta H_0^2/H^2}}{1-\beta H_0^2/H^2+2\beta^2 H_0^4/H^4}.$$

$$\left(\frac{H^2}{H_0^2}-2\beta\right)e^{\beta H_0^2/H^2} = \Omega_{\rm m0}(\beta,\Lambda)\,(1+z)^3+\Omega_{\Lambda 0},$$
$$\Omega_{\rm m0}(\beta,\Lambda) = (1-2\beta)e^{\beta}-\Omega_{\Lambda 0},$$

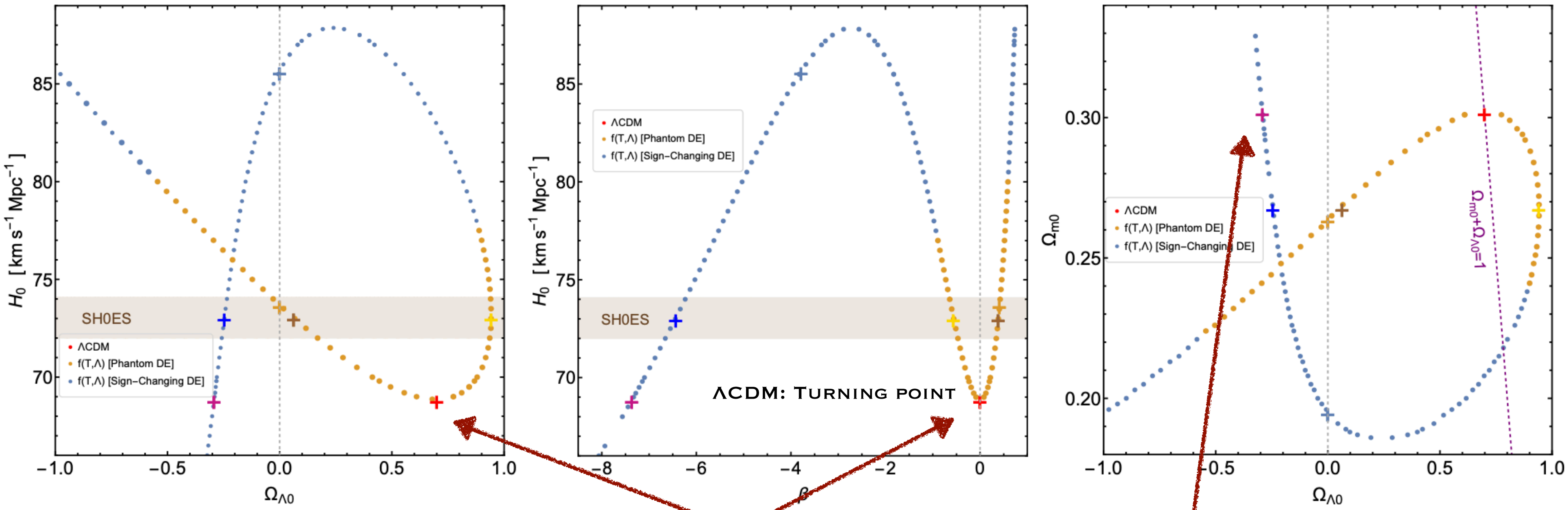


Region IV $(\beta < -1/2)$

Interpretating the DESI evidence: cosmic slow down of acceleration via f(T) gravity with β < 0 for z<1

SH0ES collaboration measurement
$H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1}
Riess et al. Astrophys. J. Lett. 934, L7 (2022) 2112.04510

Exponential infrared teleparallel model with cosmological constant



The points correspond to models having parameter sets $\{H_0, \Omega_{m0}, \Omega_{\Lambda 0}, \beta\}$ that satisfy $D_M(z_*) = 13872.83$ Mpc constrained strictly and almost model independently through the measurement of θ_* ($100\theta_* = 1.041085$) and the CMB-based constraint $\Omega_{m0}h^2 = 0.14314$ on the physical energy density.

The dotted purple line in the right panel represents $\Omega_{m0} + \Omega_{\Lambda 0} = 1$.

$\Omega_{\Lambda 0} < 0$

Remark I: Purple plus sign represents the model in line with findings from DESI paper given in Calderon, et al. 2405.04216 that adds an extensive class of model-agnostic reconstructions with acceptable fits to the data, including models where cosmic acceleration slows down at low redshifts.

Remark II: standard ΛCDM model (red plus) corresponds to a very special point, namely, to a local minimum in the left and middle panels.

Concluding remarks and future plan

- These are the First attempts on having negative energy densities in the past.
- In this model, the modified Friedmann equation does not allow us to isolate the Hubble parameter H as a function of redshift.
- Yet we are now seeing that torsional dark energy models may provide very different theoretical opportunities , such as resolving tensions, giving possible slow down in acceleration.
- We show a case study, here we should consider the two different branches of Lambert W function associated with negative and positive powers of the exponential.
- standard ΛCDM model corresponds to a very special point, turning point for H0.
- For sign-changing model, the beginning of the acceleration at an earlier time than expected results in a comoving Hubble parameter, that is in huge discrepancy with the BAO data.
- we should make sure that G/GN does not change much in the evolution of the universe. Say at most by 10% at BBN.

$$-\frac{\dot{G}_{\rm eff}}{G_{\rm eff}} = \frac{\dot{f}_T}{f_T} = \frac{f_{TT}}{f_T}\dot{T} = \frac{2(\beta H_0^2/H^2)^2}{1-\beta H_0^2/H^2}\frac{\mathrm{d}H}{\mathrm{d}\mathcal{N}},$$

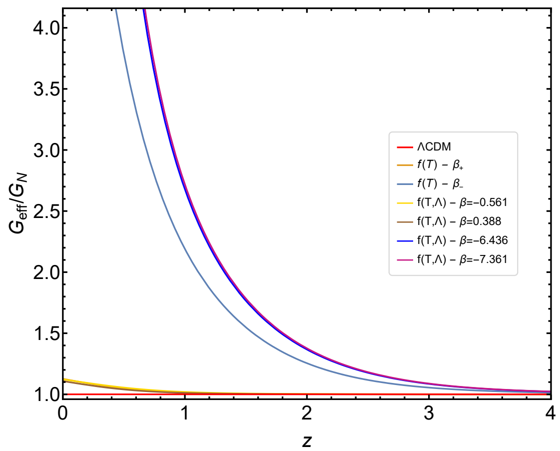


FIG. 7. $G_{\rm eff}/G_{\rm N}$ vs z

$$f(T) = Te^{\beta T_0/T} + g(T).$$

The advantage of this choice is that we can set

$$f_T(z=0) = 1 \quad \text{and} \quad f_{TT}(z=0) = 0,$$

where the first ensures $G_{\rm eff}(z=0) = G_N$ and the second ensures that $G_{\rm eff}$ varies only slowly in the late universe. To satisfy these conditions with a minimal function of T, one can introduce a quadratic correction equation for $g(T)$, which leads to

$$f(T) = Te^{\beta T_0/T} + \alpha_1 T + \alpha_2 T^2/T_0.$$

These conditions are satisfied for

$$\alpha_1 = 1 + (\beta^2 + \beta - 1)e^{\beta} \quad \text{and} \quad \alpha_2 = -\beta^2 e^{\beta}/2.$$